

## Contents

**Preface** *XI*

**List of Symbols** *XIII*

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Why Study Drying Mud?	2
1.2	Objectives and Organization of the Book	4
1.3	Approach and Scope	6
	References	7
<b>2</b>	<b>Elasticity</b>	<b>9</b>
2.1	On Springs	9
2.2	Deformation, Displacement and Strain	11
2.3	Transformations of Strains, Principal Strains and Volumetric Strain	15
2.4	Stress	17
2.5	Thermodynamics and the Work of Deformation	21
2.6	Linear Elasticity	23
2.7	Different Formulations of Linear Elasticity	26
2.8	Plane Elasticity	29
2.8.1	Plane Strain and Plane Stress	29
2.8.2	Airy Stress Function	31
2.9	Summary	32
2.10	Further Reading	33
	References	33
<b>3</b>	<b>Fracture Mechanics</b>	<b>35</b>
3.1	Griffith and Fracture Energy	35
3.2	Stress Concentration	40
3.3	Stress Intensity Factors	41
3.4	Fracture Toughness and the Relationship Between $K$ and $G$	44
3.5	Summary of the Critical Conditions for Fracture	46

3.6	An Example: Thin-Film Fracture	47
3.7	Nonlinear and Dissipative Effects of Fracture	53
3.7.1	A Plastic Zone Model of Fracture	54
3.7.2	A Mesoscopic View – The Path-Independent $J$ -integral	56
3.7.3	Dynamic Elasticity and Dynamic Fracture	58
3.8	Crack Path Selection	60
3.9	Summary and Further Reading	64
	References	65
<b>4</b>	<b>Poroelasticity</b>	<b>69</b>
4.1	Pressure and Stress in a Two-component System	70
4.1.1	Fick's Laws	73
4.1.2	Darcy's Law	74
4.1.3	Network and Total Stress	74
4.2	Linear Poroelasticity	75
4.2.1	Poroelastic Energy Density	77
4.2.2	Poroelastic Constitutive Relations	78
4.2.3	Different Formulations of Poroelasticity	82
4.3	Relationship Between Poroelasticity and Thermoelasticity	84
4.4	Worked Examples of Poroelastic Deformation	86
4.5	Poroelasticity and a Driving Force for Fracture	88
4.6	Summary and Further Reading	92
	References	94
<b>5</b>	<b>Colloids and Clays</b>	<b>97</b>
5.1	DLVO Theory	98
5.1.1	van der Waals Potential	98
5.1.2	Electrostatic Potential	101
5.1.3	DLVO Theory and its Limitations	106
5.2	Clays	110
5.3	Summary and Further Reading	114
	References	115
<b>6</b>	<b>Desiccation</b>	<b>117</b>
6.1	Surface Tension and Capillary Pressure	118
6.1.1	Contact Lines and Capillary Rise	120
6.2	Solidification Through Evaporation	122
6.2.1	Skin Formation	122
6.2.2	Crystals and Cages	124
6.2.3	Aggregation	126
6.3	Pore-Scale Processes	128
6.3.1	Structure of a Drying Soil	130
6.3.2	Dynamics of a Drying Soil	131
6.4	Continuum Models of Drying	135
6.4.1	Surface Drying	135

6.4.2	Internal Transport: Carman–Kozeny	139
6.5	Further Reading	140
	References	141
<b>7</b>	<b>Patterns of Crack Networks in Homogeneous Media</b>	<b>145</b>
7.1	Introduction	145
7.2	Experimental Observations	146
7.2.1	Sequential Fragmentation and Length Scale Selection	148
7.2.2	Scaling of Crack Width	152
7.2.3	Distribution of Angles Between Cracks	153
7.3	Directional Drying	154
7.4	Characterizing the Crack Pattern: 2D View	155
7.4.1	Scale Invariance in Crack Patterns: Self-Similar and Self-Affine Structures	155
7.4.1.1	Scale Invariant Crack Width Distribution	156
7.4.1.2	Fractal Dimension of the Crack Edge	157
7.4.1.3	Self-Affinity of the Fracture Surface	158
7.4.1.4	Fractal Fracture Mechanics	160
7.4.2	Topology and Connectivity of the Crack Network	161
7.4.2.1	Minkowski Numbers and Densities	165
7.4.2.2	Network Theory Approach: Mapping onto an Equivalent Network	167
7.4.3	Percolation	169
7.5	Instabilities: Spirals and Wavy Cracks, En Echelon/En Passant Cracks, Star Cracks, and Wing Cracks	173
7.5.1	En Echelon Cracks	174
7.5.2	En Passant Cracks	174
7.5.3	Spiral Cracks	175
7.5.4	Wavy Cracks	177
7.5.5	Star Bursts and More Patterns	178
7.6	Crack Dynamics and Branching Cracks	179
7.7	Transition Between Different Modes of Instability and Fracture	182
7.7.1	Dendrite to Fracture	182
7.7.2	Viscous Fingering to Fracture	184
7.7.3	Invasion Percolation to Fracture	185
7.8	Towards Three Dimensions: Geological Formations, Drying Soil and Peeling	188
7.8.1	Obreimoff's Experiment	188
7.8.2	Natural Mud Cracks in Quasi-2D	189
7.9	Simulation of Quasi-2D Patterns	190
7.9.1	2D Modelling of Fracture: The Fibre Bundle Model	191
7.9.2	Random Fuse Model	192
7.9.3	Spring Network Model	192
7.9.4	Other Models	196

7.10	Summary	197
7.11	Further Reading	197
	References	198
<b>8</b>	<b>The Effects of Plasticity on Crack Formation</b>	<b>207</b>
8.1	Introduction to Rheology	207
8.1.1	Elastic Material and Fluid	208
8.1.2	Linear Viscoelasticity	211
8.1.3	Bingham Model	214
8.2	Elastoplasticity for Slow Deformation Processes	216
8.2.1	Decomposition of Elastic and Plastic Deformation	216
8.2.2	Thermodynamics of Elastoplasticity	218
8.2.3	Yield Conditions and the Normality Law	219
8.2.4	Yield Conditions of Paste-Like Materials	222
8.3	Crack Propagation in a Layer of Wet Paste	223
8.3.1	Plumose Structure in Crack Surfaces	223
8.3.2	Microscopic Observation of Plastic Deformation	224
8.3.3	Measurements of the Speed of Crack Growth in a Uniform Paste Layer	227
8.4	Theoretical Approaches for Crack Velocities	230
8.4.1	Viscoelastic Effect on Crack Propagation: 1D Lattice Model of Rheological Elements	231
8.4.2	Competition of Global Plastic Relaxation and Crack Growth	233
8.5	Memory Effect of Paste Due to Its Plasticity	238
8.5.1	Memory of Vibration and Its Visualization as Desiccation Crack Pattern	239
8.5.1.1	Memory of Vibration and Lamellar Crack Pattern	239
8.5.1.2	Plasticity of Paste	241
8.5.1.3	Condition for the Memory Effect of Vibration: Experimental Results	243
8.5.2	Residual Tension Theory to Explain Memory Effect of Vibration	244
8.5.2.1	Quasi-linear Analysis	245
8.5.2.2	Governing Equations for Non-linear Analysis	249
8.5.2.3	Non-linear Analysis	251
8.5.2.4	Condition for the Memory Effect of Vibration: Theoretical Explanation	253
8.5.3	Position Control of Cracks by Memory Effect and Faraday Waves	254
8.5.4	Memory of Flow and a Role of Interaction Between Colloidal Particles	258
	Further Reading	262
	References	263

<b>9</b>	<b>Special Topics</b>	<b>267</b>
9.1	Tailoring Crack Patterns	267
9.1.1	Effect of Electric Fields on Desiccation Cracks	268
9.1.1.1	Effects of a Direct Field (DC)	268
9.1.1.2	Effect of an Alternating Field (AC)	270
9.1.1.3	DC Field Effect in Drying Droplets	271
9.1.2	Effect of a Magnetic Field on Desiccation Cracks	274
9.1.3	Patterning Cracks Through Micro-Technology	276
9.2	Designing Crack-Resistant Materials and Composites	279
9.2.1	Composites of Soft and Hard Particles	284
9.2.1.1	Employing Heterogeneous Material	285
9.2.2	Crack Reduction with 'Liquid Particles'	288
9.3	Crack Patterns in Drying Droplets of Biofluids	290
9.3.1	Human Blood Droplets and Drying Dynamics	291
9.3.2	Effect of Relative Humidity on Drying Droplets	295
9.3.3	Substrate Effect on Drying Droplets of Blood	296
9.4	Evolving Crack Networks	297
9.4.1	Columnar Joints	298
9.4.2	Evolving Mud Cracks	304
9.4.3	Other Crack Patterns	308
9.5	Further Reading	310
	References	311
<b>Appendix A:</b>	<b>A Primer on Vectors and Tensors</b>	<b>317</b>
A.1	Tensor Notation	317
A.2	Tensor Multiplication	319
A.3	Tensor Transformations	321
A.4	Tensor Differentiation	323
<b>Appendix B:</b>	<b>Fractals: Self-Similar and Self-Affine Systems</b>	<b>327</b>
B.1	Self-Similarity and Fractal Dimension	327
B.2	Self-Affine Systems	331
B.3	Further Reading	332
	References	333
<b>Appendix C:</b>	<b>Formulation of Elastoplasticity Based on Dissipation Functions</b>	<b>335</b>
	References	336
<b>Appendix D:</b>	<b>Steady Propagating Solution of Langer Model</b>	<b>337</b>
<b>Appendix E:</b>	<b>Stress Expression in Finite Deformation Theory</b>	<b>339</b>
	References	341
	<b>Index</b>	<b>343</b>