

Contents

1	Lévy Processes and Applications	1
1.1	Lévy Processes and Infinite Divisibility	1
1.2	Some Examples of Lévy Processes	5
1.3	Lévy Processes and Some Applied Probability Models	14
	Exercises	27
2	The Lévy–Itô Decomposition and Path Structure	35
2.1	The Lévy–Itô Decomposition	35
2.2	Poisson Random Measures	37
2.3	Functionals of Poisson Random Measures	42
2.4	Square-Integrable Martingales	46
2.5	Proof of the Lévy–Itô Decomposition	53
2.6	Lévy Processes Distinguished by Their Path Type	55
2.7	Interpretations of the Lévy–Itô Decomposition	58
	Exercises	64
3	More Distributional and Path-Related Properties	71
3.1	The Strong Markov Property	71
3.2	Duality	77
3.3	Exponential Moments and Martingales	78
	Exercises	87
4	General Storage Models and Paths of Bounded Variation	91
4.1	General Storage Models	91
4.2	Idle Times	92
4.3	Change of Variable and Compensation Formulae	94
4.4	The Kella–Whitt Martingale	102
4.5	Stationary Distribution of the Workload	104
4.6	Small-Time Behaviour and the Pollaczek–Khintchine Formula	106
	Exercises	110

5	Subordinators at First Passage and Renewal Measures	115
5.1	Killed Subordinators and Renewal Measures	115
5.2	Overshoots and Undershoots	123
5.3	Creeping	125
5.4	Regular Variation and Tauberian Theorems	130
5.5	Dynkin–Lamperti Asymptotics	135
5.6	Special and Complete Subordinators	138
	Exercises	146
6	The Wiener–Hopf Factorisation	153
6.1	Local Time at the Maximum	154
6.2	The Ladder Process	160
6.3	Excursions	167
6.4	The Wiener–Hopf Factorisation	170
6.5	Examples of the Wiener–Hopf Factorisation	182
6.6	Vigon’s Theory of Philanthropy and More Examples	188
6.7	Brief Remarks on the Term “Wiener–Hopf”	192
	Exercises	192
7	Lévy Processes at First Passage	197
7.1	Drifting and Oscillating	197
7.2	Cramér’s Estimate	203
7.3	A Quintuple Law at First Passage	207
7.4	The Jump Measure of the Ascending Ladder Height Process	213
7.5	Creeping	214
7.6	Regular Variation and Infinite Divisibility	217
7.7	Asymptotic Behaviour at First Passage	221
	Exercises	223
8	Exit Problems for Spectrally Negative Processes	231
8.1	Basic Properties Reviewed	231
8.2	The One-Sided and Two-Sided Exit Problems	234
8.3	The Scale Functions $W^{(q)}$ and $Z^{(q)}$	240
8.4	Potential Measures	244
8.5	Identities for Reflected Processes	247
	Exercises	250
9	More on Scale Functions	257
9.1	The Wiener–Hopf Factorisation Revisited	257
9.2	Scale Functions and Philanthropy	259
9.3	Special and Conjugate Scale Functions	263
9.4	Tilting and Parent Processes Drifting to $-\infty$	265
9.5	Complete Scale Functions	267
	Exercises	269

10	Ruin Problems and Gerber–Shiu Theory	275
	10.1 Review of Distributional Properties at Ruin	276
	10.2 The Gerber–Shiu Measure	278
	10.3 Reflection Strategies	281
	10.4 Refraction Strategies	285
	10.5 Perturbed Processes and Tax	296
	Exercises	302
11	Applications to Optimal Stopping Problems	307
	11.1 Sufficient Conditions for Optimality	307
	11.2 The McKean Optimal Stopping Problem	309
	11.3 Smooth Fit Versus Continuous Fit	314
	11.4 The Novikov–Shiryaev Optimal Stopping Problem	318
	11.5 The Shepp–Shiryaev Optimal Stopping Problem	324
	Exercises	331
12	Continuous-State Branching Processes	335
	12.1 The Lamperti Transform	335
	12.2 Long-Term Behaviour	338
	12.3 Conditioned Processes and Immigration	345
	12.4 Concluding Remarks	356
	Exercises	356
13	Positive Self-similar Markov Processes	363
	13.1 Definition and Examples	364
	13.2 Conditioned Processes and Self-similarity	367
	13.3 The Second Lamperti Transform	372
	13.4 Lamperti-Stable Processes	380
	13.5 Self-similar Continuous-State Branching Processes	390
	13.6 Entrance Laws and Recurrent Extensions	392
	13.7 Spectrally Negative Processes	396
	Exercises	406
	Epilogue	411
	Hints for Exercises	415
	Chapter 1	415
	Chapter 2	418
	Chapter 3	420
	Chapter 4	421
	Chapter 5	423
	Chapter 6	425
	Chapter 7	427
	Chapter 8	429
	Chapter 9	430
	Chapter 10	431
	Chapter 11	432

Chapter 12	433
Chapter 13	435
References	437
Index	451